M2 Internship – ENS Lyon
Scheduling series parallel graphs to minimize the makespan

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Feb – Jul 2014

Outline

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Introduction

Motivation

- factorisation of sparse matrices creates task trees to be scheduled
- need to schedule such trees using parallelisation between the nodes and inside each node

Related work

- Prasanna, Musicus (1996)
- Beaumont, Guermouche (2007)
- Marchal, Sinnen, Vivien (2013)

Goals

- reprove the results of Prasanna & Musicus using scheduling arguments
- take in account the memory constraints

Outline

Model

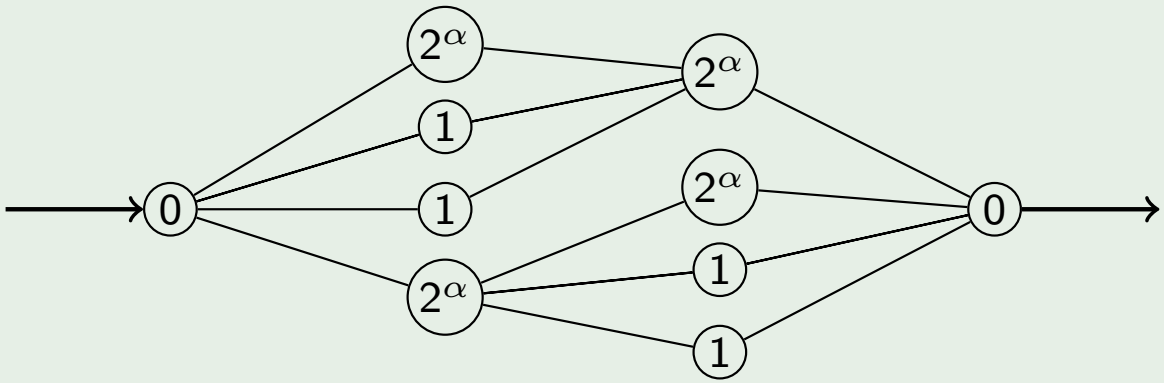
Goal

Schedule a series-parallel graph (generalisation of trees) G under p processors minimizing the makespan

Hypotheses

- Task of length L is completed with p processors in time L/p^{α} with $0 < \alpha < 1$
- A non-integer share of processors can be allocated to a task
- We can allocate less than 1 processor to a task
- The tasks are malleable: the share of processors can be modified

Example



Theorem to prove

Theorem (Prasanna & Musicus)

For any series-parallel graph G, any optimal schedule under a constant number of processors p respects:

- *each task is assigned a constant processing power*
- **Processor Flow Conservation property:** *every 'siblings' terminate at the same time and all the processing power is then assigned to the parent(s)*
- *the allocation for different values of p are proportionals*

Corollary

Under a constant number of processors, each graph G has the same optimal makespan than the task T_G of length L_G.

For series: $L_G = L_1 + L_2$ *For parallel:* $L_G = \left(L_1^{1/\alpha} + L_2^{1/\alpha}\right)^{\alpha}$

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First lemmas used in the proof

Lemma

An allocation minimizing the makespan uses at any time all the processors.

Definition (Clean interval)

An interval during which no task terminates in the considered schedule.

Lemma

We have to process n independent tasks in parallel with a constant number of processors p. A schedule that does not allocate a constant number of processors per task on clean intervals is not optimal.

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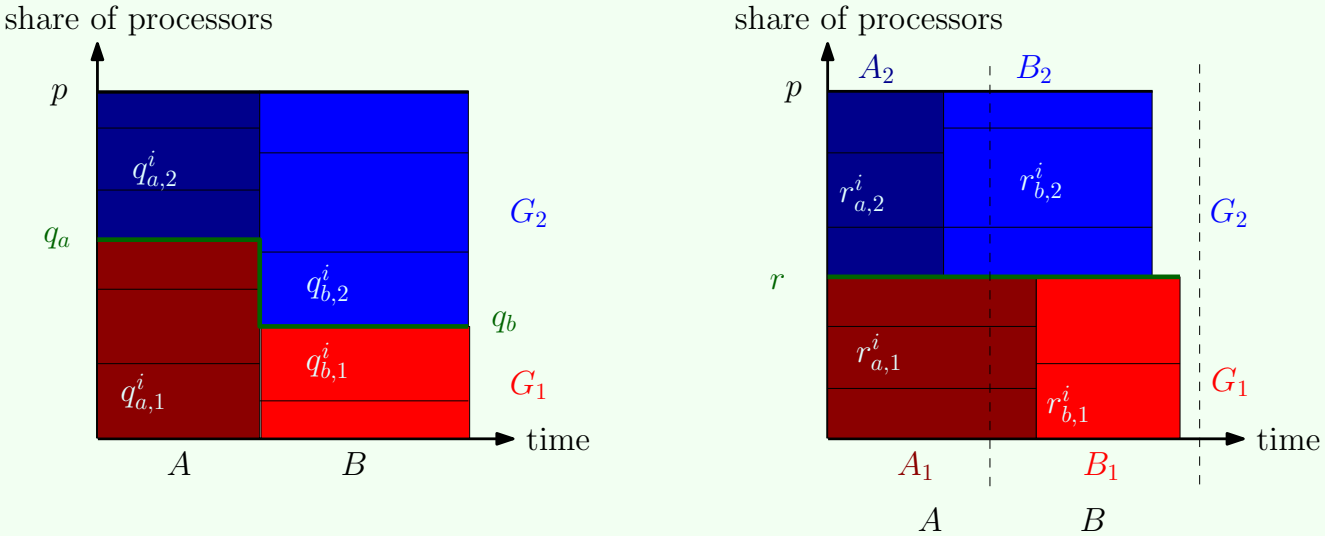
Lemma

Let G be the graph given by the parallel composition of G_1 and G_2. If a constant number of processors is given to schedule G, then any optimal schedule associates a constant number of processors to G_1 and to G_2.

Proof.



Property used: $(ab)^\alpha = a^\alpha b^\alpha$ □

Remark

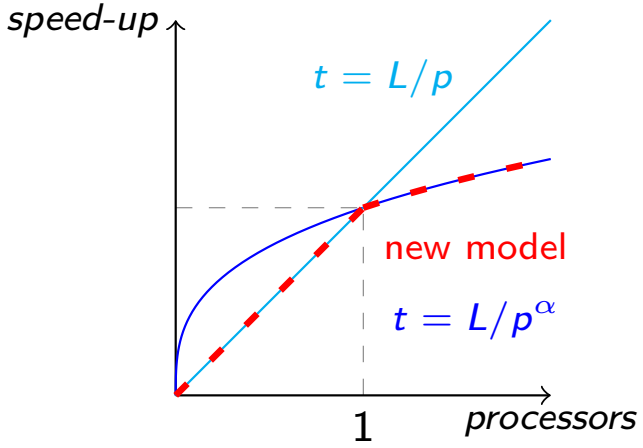
Consequently, G_1 and G_2 have the same makespan.

Outline

Introduction to the model

Modification

- $p \geq 1$: the time to complete a task L is L/p^{α}
- $p \leq 1$: the time to complete a task L is L/p



Time to complete a task in function of the processing power in the different models

Consequences

The last lemma does not hold so the previous theorem does not hold.
We cannot compute the optimal schedule.

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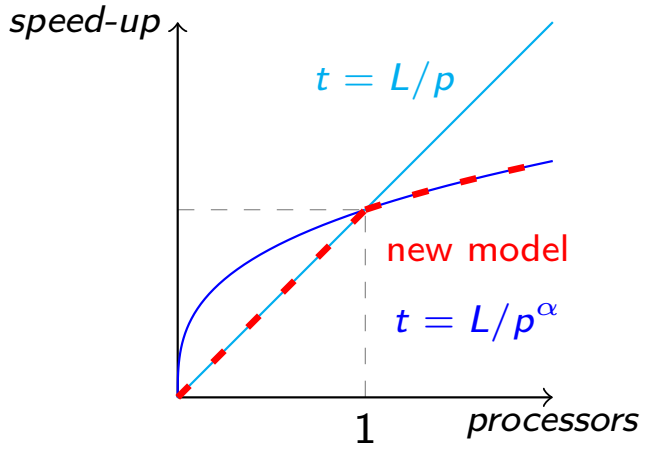

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Motivation

Definition (PM allocation)

The allocation that associates to each task the share computed by the formulas of previous section.

Theorem

No PM algorithm is a constant ratio approximation.

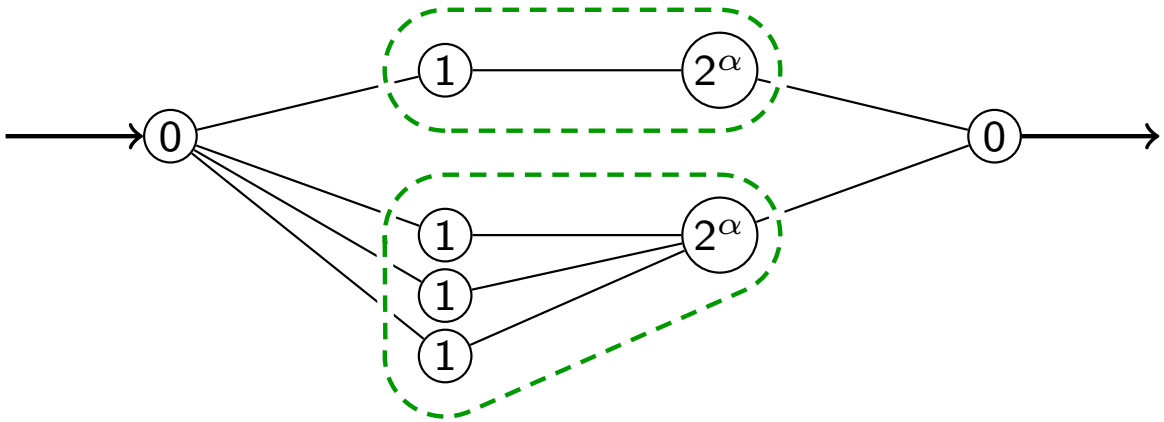
PFC schedules

Definition (PFC schedule)

An allocation that associates a constant share to each subgraph at every parallel node, under a constant number of processors.

Theorem

The optimal PFC allocation is not always the optimal schedule.
This statement is still valid on tree-shaped graphs and for moldable tasks.



Example of tree-shaped graph illustrating the theorem, with $p = 4$

PFC schedules

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Remark (Optimal PFC allocation seen as an approximation)

The approximation ratio is smaller than $p^{1-\alpha}$.
The worst ratio observed is 1.1*: the exact ratio is unknown.*

Remark

We can easily check that a PFC allocation is optimal among PFC's: at every parallel nodes, the two branches must terminate at the same time.
But we cannot compute the optimal PFC.

Heuristic

New goal

Compute the makespan-optimal PFC schedule.
No exact algorithm is known, so use of a heuristic.

Idea

- In PM schedules: the makespan of tasks with less than 1 processors is underestimated
- Artificially increase the need of processors: increase the length L seen by the algorithm
- Goal: $L/p = \bar{L}/p^{\alpha} \Rightarrow \bar{L} := L \cdot p^{\alpha-1} > L$
- Algorithm: compute the PM schedule, modify the L_i, and iterate

Remark

- *For any graph G scheduled, define A_G the largest difference of makespan between two parallel branches*
- *A_G is a way to check the schedule returned by the heuristic*

Results of the heuristic

Results proved

- on any two-tasks graph G, there exists $\alpha_G < 0.5$ such that for any $\alpha > \alpha_G$, the heuristic converges
- for $\alpha > \alpha_G$, the odd and even subsequences of L_i are monotone and adjacent
- for $\alpha < \alpha_G$, the heuristic does not converge

Observations on random/selected graphs

- for any graph G, for any $\alpha > 0.5$, the heuristic converges
- $A_{G^{2j}}$ and $A_{G^{2j+1}}$ decrease and converge to 0

Outline

Introduction

Assumptions

- each output file is of size $f_i = 1$
- each task has an internal memory need of $n_i = 0$
- to execute a task, the output files of the children and of the task must be allocated
- the lengths of the tasks are $w_i \in \{0, 1\}$ (unique difference with Pebble Game)
- the time to complete a task of size w_i with p processors is always w_i/p^{α}

Lemma (backbone of the following theorems)

Under the hypotheses:

- *tn independent tasks of length 1*
- *tp processors*
- *memory constraints forbid to parallelize more than t tasks*

optimality $\Leftrightarrow$ batches of t tasks

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optimality $\Leftrightarrow$ *batches of t tasks*

NP-completeness of the BiObjective problem

The BiObjectiveParallelTreeScheduling problem

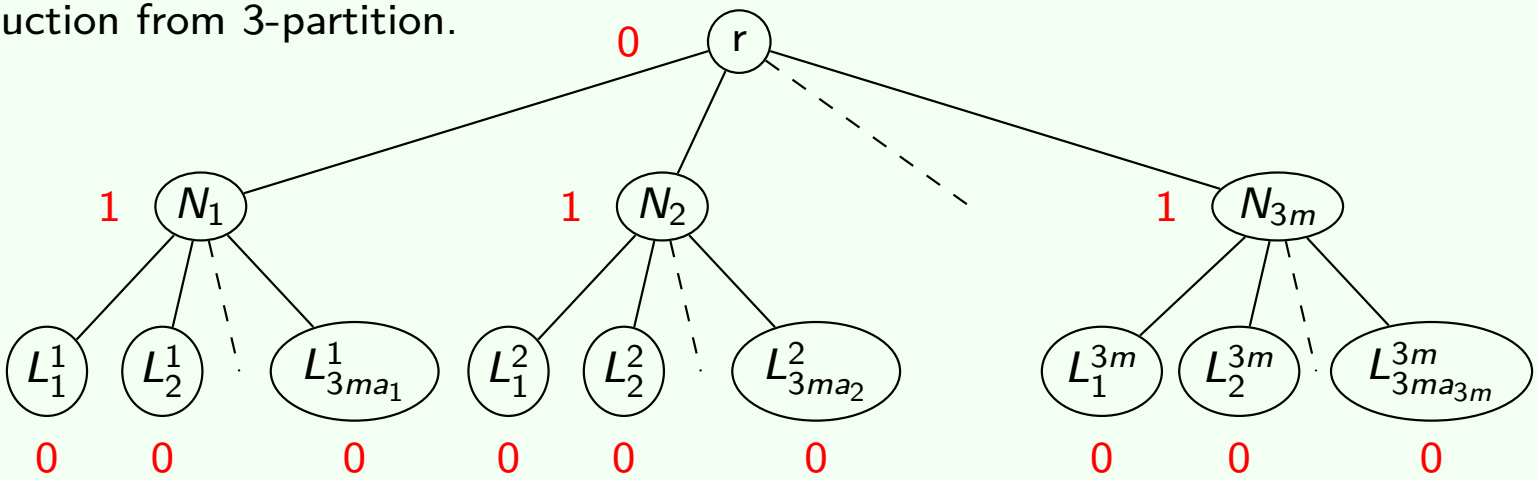
Given a tree-shaped task graph T provided with memory weights and task durations, p processors, and two bounds $B_{C_{max}}$ and B_{mem}, is there a schedule of the task graph on the processors whose makespan is not larger than $B_{C_{max}}$ and whose peak memory is not larger than B_{mem}?

Theorem

The BiObjectiveParallelTreeScheduling problem is NP-Complete.

Proof.

Reduction from 3-partition.



Inapproximation results

Theorem

There is no algorithm $\mathcal{A}$ that is both a β-approximation for makespan minimization and a γ-approximation for memory peak minimization when scheduling in-tree task graphs.

Theorem

When scheduling in-tree task graphs with p processors, there is no algorithm $\mathcal{A}$ that is both a $\beta(p)$-approximation for makespan minimization and a $\gamma(p)$-approximation for memory peak minimization with $\beta(p)$ and $\gamma(p)$ verifying:

$$\gamma(p)\beta(p)^{1-\alpha} > \left(\frac{p}{\log p + 1}\right)^{1-\alpha}$$

Outline

Conclusion

Model $T = L/p^{\alpha}$ for all p

- Every graph is equivalent to a task, which length is computable efficiently

Model $T = L/p$ for $p < 1$

- PM schedules are not λ-approximations
- PFC schedules are not optimal
- A heuristic probably converges towards the PFC optimal schedule
- The approximation ratio is not known

Memory-aware model

- Deciding if there exists a schedules that respects a makespan and a memory constraint is NP-complete
- There is a bound over the approximate ratios

Future work

Model $T = L/p$ for $p < 1$

- Compute the approximation ratio of the optimal PFC allocation
- (Dis)prove the convergence of the heuristic

Memory-aware model

- Design a heuristic with guaranties on makespan and memory peak (in progress)